

CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

1 Marks Questions

1. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Ans. The binomial of degree 35 can be $x^{35} + 9$.

The binomial of degree 100 can be t^{100} .

2. Which of the following expression is a polynomial

(a) $x^3 - 1$

(b) $\sqrt{x} + 2$

(c) $x^2 - \frac{1}{x^2}$

(d) $\sqrt{t} + 5t - 1$

Ans. (a) $x^3 - 1$

3. A polynomial of degree 3 in x has at most

(a) 5 terms

(b) 3 terms

(c) 4 terms

(d) 1 term

Ans. (b) 3 terms



4. The coefficient of x^2 in the polynomial $2x^3 + 4x^2 + 3x + 1$ is

(a) 2

(b) 3

(c) 1

(d) 4

Ans. (d) 4

5. The monomial of degree 50 is

(a) $x^{50} + 1$

(b) $2x^{50}$

(c) $x+50$

(d) 50

Ans. (b) $2x^{50}$

6. Divide $f(x)$ by $g(x)$ and verify the remainder $f(x) = x^3 + 4x^2 - 3x - 10$, $g(x) = x + 4$

Ans. Dividend = $x^3 + 4x^2 - 3x - 10$, divisor = $x + 4$

Quotient = $x^2 - 3$, Remainder = 2

Dividend = Divisor \times quotient + Remainder

$$= (x + 4) (x^2 - 3) + 2$$

$$= x^3 - 3x + 4x^2 - 12 + 2$$

$$= x^3 + 4x^2 - 3x - 10$$

7. Which of the following expression is a monomial

- (a) $3 + x$
- (b) $4x^3$
- (c) $x^6 + 2x^2 + 2$
- (d) None of these

Ans. (a) $3 + x$

8. A linear polynomial

- (a) May have one zero
- (b) has one and only one zero
- (c) May have two zero
- (d) May have more than one zero

Ans. (b) has one and only one zero

9. If $P(x) = x^3 - 1$, then the value of $P(1) + P(-1)$ is

- (a) 0
- (b) 1
- (c) 2
- (d) -2

Ans. (d) -2

10. when polynomial $x^3 + 3x^2 + 3x + 1$ is divided by $x + 1$, the remainder is

- (a) 1



(b) 0

(c) 8

(d) - 6

Ans. (b) -6

11. Factorise $x^2 + y - xy - x$

Ans. $x^2 + y - xy - x$

$$x^2 - x + y - xy = x^2 - x - xy + y$$

$$= x(x - 1) - y(x - 1)$$

$$= (x - 1)(x - y)$$

12. The value of K for which $x - 1$ is a factor of the polynomial $4x^3 + 3x^2 - 4x + K$ is

(a) 0

(b) 3

(c) - 3

(d) 1

Ans. (c) - 3

13. The factors of $12x^2 - x - 6$ are

(a) $(3x - 2)(4x + 3)$

(b) $(12x + 1)(x - 6)$

(c) $(12x - 1)(x + 6)$

(d) $(3x + 2)(4x - 3)$

Ans. (d) $(3x + 2)(4x - 3)$

14. $x^3 + y^3 + z^3 - 3xyz$ is

(a) $(x + y - z)^3$

(b) $(x - y + z)^3$

(c) $(x + y + z)^3 - 3xyz$

(d) $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Ans. (d) $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

15. The expanded form of $(x + y - z)^2$ is

(a) $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(b) $x^2 + y^2 - z^2 + 2xy - 2yz - 2xz$

(c) $x^2 + y^2 + z^2 + 2xy - 2yz - 2zx$

(d) $x^2 + y^2 + z^2 + 2xy + 2yx + 2xz$

Ans. (c) $x^2 + y^2 + z^2 + 2xy - 2yz - 2zx$

16. Find the integral zeroes of the polynomial $x^3 + 3x^2 - x - 3$

Ans. Given polynomial $P(x) = x^3 + 3x^2 - x - 3$

$$p(x) = x^2(x + 3) - 1(x + 3)$$

$$= (x + 3)(x^2 - 1)$$

For zeros $p(x) = 0$

$$(x+3)(x^2-1)=0$$

$$(x+3)(x+1)(x-1)=0$$

$$x=-3, x=-1, x=1$$

Zeros of polynomial -1, 1, and -3.

17. The value of $(102)^3$ is

(a) 1061208

(b) 1001208

(c) 1820058

(d) none of these

Ans. (a) 1061208

18. $(a-b)^3 + (b-c)^3 + (c-a)^3$ is equal to

(a) $3abc$

(b) $3(a-b)(b-c)(c-a)$

(c) $3a^3b^3c^3$

(d) $[a-(b+c)]^3$

Ans. (b) $3(a-b)(b-c)(c-a)$

19. The zeroes of the polynomial $p(x) = x(x-2)(x+3)$ are

(a) 0

(b) 0, 2, 3

(c) 0, 2, -3

(d) none of these

Ans. (c) 0, 2, -3

20. If $(x+1)$ and $(x-1)$ are factors of Px^3+x^2-2x+9 then value of p and q are

(a) $p = -1, q = 2$

(b) $p = 2, q = -1$

(c) $p = 2, q = 1$

(d) $p = -2, q = -2$

Ans. (b) $p = 2, q = -1$

21. If $x+y+z = 0$, then $x^3 + y^3 + z^3$ is

(a) xyz

(b) $2xyz$

(c) $3xyz$

(d) 0

Ans. (b) $2xyz$

22. The value of $(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c)$ when $a + b + c = 3x$, is

(a) 3

(b) 2

(c) 1

(d) 0

Ans. (c) 1

23. Factors of $x^2 + 3\sqrt{2}x + 4$ are

(a) $(x + 2\sqrt{2})(x - \sqrt{2})$

(b) $(x + 2\sqrt{2})(x + \sqrt{2})$

(c) $(x - 2\sqrt{2})(x + \sqrt{2})$

(d) $(x - 2\sqrt{2})(x - \sqrt{2})$

Ans. (b) $(x + 2\sqrt{2})(x + \sqrt{2})$

24. The degree of constant function is

(a) 1

(b) 2

(c) 3

(d) 0

Ans. (d) 0



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2 Marks Questions

1. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$

(iv) $\sqrt{2}x - 1$

Ans. (i) $2 + x^2 + x$

The coefficient of x^2 in the polynomial $2 + x^2 + x$ is 1.

(ii) $2 - x^2 + x^3$

The coefficient of x^2 in the polynomial $2 - x^2 + x^3$ is -1 .

(iii) $\frac{\pi}{2}x^2 + x$

The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x - 1$

The coefficient of x^2 in the polynomial $\sqrt{2}x - 1$ is 0.

2. Find the value of the polynomial $5x - 4x^2 + 3$ at



(i) $x = 0$

(ii) $x = -1$

(iii) $x = 2$

Ans. (i) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute 0 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get

$$f(0) = 5(0) - 4(0)^2 + 3$$

$$= 0 - 0 + 3$$

$$= 3$$

Therefore, we conclude that at $x = 0$, the value of the polynomial $5x - 4x^2 + 3$ is 3.

(ii) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute -1 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get

$$f(-1) = 5(-1) - 4(-1)^2 + 3$$

$$= -5 - 4 + 3$$

$$= -6$$

Therefore, we conclude that at $x = -1$, the value of the polynomial $5x - 4x^2 + 3$ is -6

(iii) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute 0 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get

$$f(2) = 5(2) - 4(2)^2 + 3$$

$$= 10 - 16 + 3$$

$$= -3$$

Therefore, we conclude that at $x = 2$, the value of the polynomial $5x - 4x^2 + 3$ is -3 .

3. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Ans. We need to find the zero of the polynomial $x - a$.

$$x - a = 0$$

$$\Rightarrow x = a$$

While applying the remainder theorem, we need to put the zero of the polynomial $x - a$ in the polynomial $x^3 - ax^2 + 6x - a$, to get

$$p(x) = x^3 - ax^2 + 6x - a$$

$$p(a) = (a)^3 - a(a)^2 + 6(a) - a$$

$$= a^3 - a^3 + 6a - a$$

$$= 5a$$

Therefore, we conclude that on dividing the polynomial $x^3 - ax^2 + 6x - a$ by $x - a$, we will get the remainder as $5a$.

4. Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 98×96

(iii) 104×96

Ans. (i) 103×107

$$103 \times 107 \text{ can also be written as } (100 + 3)(100 + 7).$$

We can observe that we can apply the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(100 + 3)(100 + 7) = (100)^2 + (3 + 7)(100) + 3 \times 7$$

$$= 10000 + 1000 + 21$$

$$= 11021$$

Therefore, we conclude that the value of the product 103×107 is 11021.

(ii) 95×96

95×96 can also be written as $(100 - 5)(100 - 4)$

We can observe that we can apply the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(100 - 5)(100 - 4) = (100)^2 + [(-5) + (-4)](100) + (-5) \times (-4)$$

$$= 10000 - 900 + 20$$

$$= 9120$$

Therefore, we conclude that the value of the product 95×96 is 9120.

(iii) 104×96

104×96 can also be written as $(100 + 4)(100 - 4)$.

We can observe that, we can apply the identity $(x + y)(x - y) = x^2 - y^2$ with respect to the expression $(100 + 4)(100 - 4)$, to get

$$(100 + 4)(100 - 4) = (100)^2 - (4)^2$$

$$= 10000 - 16$$

$$= 9984$$

Therefore, we conclude that the value of the product 104×96 is 9984.

5. Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Ans. (i) $9x^2 + 6xy + y^2$

$$9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2$$

We can observe that we can apply the identity $(x+y)^2 = x^2 + 2xy + y^2$

$$\Rightarrow (3x)^2 + 2 \times 3x \times y + (y)^2 = (3x+y)^2.$$

(ii) $4y^2 - 4y + 1$

$$4y^2 - 4y + 1 = (2y)^2 - 2 \times 2y \times 1 + (1)^2$$

We can observe that we can apply the identity $(x-y)^2 = x^2 - 2xy + y^2$

$$\Rightarrow (2y)^2 - 2 \times 2y \times 1 + (1)^2 = (2y-1)^2.$$

(iii) $x^2 - \frac{y^2}{100}$

We can observe that we can apply the identity $(x)^2 - (y)^2 = (x+y)(x-y)$

$$\Rightarrow (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right).$$

6. Verify:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Ans. (i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\Rightarrow x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

$$= (x+y) \left[(x+y)^2 - 3xy \right]$$

$$\because \text{We know that } (x+y)^2 = x^2 + 2xy + y^2$$

$$\therefore x^3 + y^3 = (x+y)(x^2 + 2xy + y^2 - 3xy)$$

$$= (x+y)(x^2 - xy + y^2)$$

Therefore, the desired result has been verified.

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$$

$$= (x-y) \left[(x-y)^2 + 3xy \right]$$

$$\because \text{We know that } (x-y)^2 = x^2 - 2xy + y^2$$

$$\therefore x^3 - y^3 = (x-y)(x^2 - 2xy + y^2 + 3xy)$$

$$= (x-y)(x^2 + xy + y^2)$$

Therefore, the desired result has been verified.

7. Factorize:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Ans.

(i) $27y^3 + 125z^3$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.

$$\begin{aligned} \cdot (3y)^3 + (5z)^3 &= (3y + 5z) \left[(3y)^2 - 3y \times 5z + (5z)^2 \right] \\ &= (3y + 5z)(9y^2 - 15yz + 25z^2). \end{aligned}$$

(ii) $64m^3 - 343n^3$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

$$\begin{aligned} \cdot (4m)^3 - (7n)^3 &= (4m - 7n) \left[(4m)^2 + 4m \times 7n + (7n)^2 \right] \\ &= (4m - 7n)(16m^2 + 28mn + 49n^2) \end{aligned}$$

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m - 7n)(16m^2 + 28mn + 49n^2)$.

8. Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Ans. The expression $27x^3 + y^3 + z^3 - 9xyz$ can also be written as

$$(3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z.$$

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

$$\begin{aligned}
&\therefore (3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z \\
&= (3x + y + z) \left[(3x)^2 + (y)^2 + (z)^2 - 3x \times y - y \times z - z \times 3x \right] \\
&= (3x + y + z) (9x^2 + y^2 + z^2 - 3xy - yz - 3xz).
\end{aligned}$$

Therefore, we conclude that after factorizing the expression $27x^3 + y^3 + z^3 - 9xyz$, we get $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$.

9. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z) \left[(x - y)^2 + (y - z)^2 + (z - x)^2 \right]$

Ans.

LHS is $x^3 + y^3 + z^3 - 3xyz$ and RHS is $\frac{1}{2}(x + y + z) \left[(x - y)^2 + (y - z)^2 + (z - x)^2 \right]$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

And also, we know that $(x - y)^2 = x^2 - 2xy + y^2$.

$$\begin{aligned}
&\frac{1}{2}(x + y + z) \left[(x - y)^2 + (y - z)^2 + (z - x)^2 \right] \\
&\frac{1}{2}(x + y + z) \left[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2) \right] \\
&\frac{1}{2}(x + y + z) (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx) \\
&(x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx).
\end{aligned}$$

Therefore, we can conclude that the desired result is verified

10. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 0$.

Ans. We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

We need to substitute $x^3 + y^3 + z^3 = 0$ in

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx), \text{ to get}$$

$$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - zx), \text{ or}$$

$$x^3 + y^3 + z^3 - 3xyz = 0$$

$$\Rightarrow x^3 + y^3 + z^3 = 3xyz.$$

Therefore, the desired result is verified

11. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Ans. (i) $(-12)^3 + (7)^3 + (5)^3$

Let $a = -12$, $b = 7$ and $c = 5$

We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Here, $a + b + c = -12 + 7 + 5 = 0$

$$\therefore (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$$

$$= -1260$$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $a = 28$, $b = -15$ and $c = -13$

We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Here, $a + b + c = 28 - 15 - 13 = 0$

$$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$

$$= 16380$$

12. Find the value of K if $x - 2$ is factor of $4x^3 + 3x^2 - 4x + K$

Ans. $x - 2$ is factor of $4x^3 + 3x^2 - 4x + K$

$$x - 2 = 0$$

$$\Rightarrow x = 2$$

$$\therefore 4(2)^3 + 3(2)^2 - 4 \times 2 + k = 0$$

$$32 + 12 - 8 + k = 0$$

$$44 - 8 + k = 0$$

$$36 + k = 0$$

$$K = -36$$

13. Factorise the polynomial $x^3 + 8y^3 + 64z^3 - 24xyz$

Ans. $x^3 + 8y^3 + 64z^3 - 24xyz$

$$x^3 + (2y)^3 + (4z)^3 - 3 \times x \times (2y) \times (4z)$$

$$= (x + 2y + 4z)[x^2 + (2y)^2 + (4z)^2 - x \times 2y - 2y \times 4z - x \times 4z]$$

$$= (x + 2y + 4z)(x^2 + 4y^2 + 16z^2 - 2xy - 8yz - 4xz)$$

14. Without actually Calculating the cubes, find the value of $(-12)^3 + (7)^3 + (5)^3$

Ans. $a^3 + b^3 + c^3 = 3abc$

if $a+b+c=0$

$$(-12)^3 + (7)^3 + (5)^3 = 3 \times -12 \times 7 \times 5$$

$$= -1260$$

$$\therefore -12 + 7 + 5 = -12 + 12 = 0$$

15. If $x-3$ and $x-\frac{1}{3}$ are both factors of px^2+5x+r , then show that $p=r$

Ans. $\because x-3$ and $x-\frac{1}{3}$ are factors of $px^2+5x+r \therefore x=3, x=\frac{1}{3}$

zero of px^2+5x+r

$$\therefore p(3)^2 + 5 \times 3 + r = 0$$

$$9p + 15 + r = 0$$

$$9p + r = -15 \text{-----(1)}$$

$$p\left(\frac{1}{3}\right)^2 + 5 \times \frac{1}{3} + r = 0$$

$$\frac{p}{9} + \frac{5}{3} + r = 0$$

$$\frac{p+15+9r}{9} = 0$$

$$p+9r = -15 \text{-----(2)}$$

$$9p+r = p+9r$$

From (1) and (2),

$$9p+r=p+9r$$

$$9p - p = 9r - r$$

$$8p = 8r$$

$$p = r$$

Hence prove.

16. Show that 5 is a zero of polynomial $2x^3 - 7x^2 - 16x + 5$

Ans. Put $x = 5$ in $2x^3 - 7x^2 - 16x + 5$

$$2 \times 5^3 - 7 \times 5^2 - 16 \times 5 + 5$$

$$= 250 - 175 - 80 + 5$$

$$= 255 - 255 = 0$$

$\therefore x = 5$ is zero of polynomial $2x^3 - 7x^2 - 16x + 5$

17. Using remainder theorem find the remainder when $f(x)$ is divided by $g(x)$

$$f(x) = x^{24} - x^{19} - 2 \quad g(x) = x + 1$$

Ans. When $f(x)$ is divided by $g(x)$

Then remainder $f(-1)$

$$F(-1) = (-1)^{24} - (-1)^{19} - 2 = 1 - (-1) - 2$$

$$= 1 + 1 - 2 = 0$$

18. Find K if $x + 1$ is a factor of $P(x) = Kx^2 - x + 2$

Ans. Here $P(x) = Kx^2 - x + 2$

$\therefore x + 1$ is factor of $P(x)$

$$\therefore P(-1) = 0$$

$$K(-1)^2 - \sqrt{2}(-1) + 2 = 0$$

$$K + \sqrt{2} + 2 = 0$$

$$K = -(2 + \sqrt{2})$$

19. Find the values of m and n if the polynomial $2x^3 + mx^2 + nx - 14$ has $x - 1$ and $x + 2$ as its factors.

Ans. $x - 1$ and $x + 2$ are factor of $2x^3 + mx^2 + nx - 14$

$$x = 1, x = -2$$

$$\therefore 2(1)^3 + m(1)^2 + n(1) - 14 = 0$$

$$2 + m + n - 14 = 0$$

$$m + n - 12 = 0$$

$$m + n = 12 \text{-----(1)}$$

$$2(2)^3 + m(2)^2 + n(2) - 14 = 0$$

$$16 + 4m + 2n - 14 = 0$$

$$4m + 2n - 2 = 0$$

$$4m + 2n = 2$$

$$2m + n = 1 \text{-----(2)}$$

Subtracting (2) from (1)

$$-m = 13 \Rightarrow m = -13$$

Put $m = -13$ in (1)

$$-13 + n = 12$$

$$N=12+13=25$$

20. Check whether $7+3x$ is a factor of $3x^2 + 7x$

Ans. Let $p(x) = 3x^2 + 7x$

$7+3x$ is factor of $p(x)$

Remainder = 0

$$\text{Remainder} = P\left(-\frac{7}{3}\right)$$

$$= 3\left(-\frac{7}{3}\right)^2 + 7\left(-\frac{7}{3}\right)$$

$$= \cancel{3} \times \frac{49}{\cancel{3}} - \frac{49}{3}$$

$$= 0$$

Hence $7+3x$ is factor of $p(x)$

21. Factorise $\frac{3}{2}x^2 - x - \frac{4}{3}$

Ans. $\frac{3}{2}x^2 - x - \frac{4}{3}$

$$\frac{3}{2} \times \frac{-4}{3} = -2$$

We factorise by splitting middle term

$$-2+1=-1$$

$$\frac{3}{2}x^2 - 2x + x - \frac{4}{3}$$

$$= \frac{3}{2}x \left(x - \frac{4}{3} \right) + 1 \left(x - \frac{4}{3} \right)$$

$$= \left(\frac{3}{2}x + 1 \right) \left(x - \frac{4}{3} \right)$$

22. Evaluate $(101)^2$ by using suitable identity

Ans. $(101)^2 = (100 + 1)^2$

$$(a + b)^2 = a^2 + 2ab + b^2$$

her $a = 100$, $b = 1$

$$(101)^2 = (100 + 1)^2 = 100^2 + 2 \times 100 \times 1 + 1^2$$

$$= 10000 + 200 + 1$$

$$= 10201$$

23. Find m and n if $x - 1$ and $x - 2$ exactly divide the polynomial $x^3 + mx^2 - nx + 10$

Ans. Let $p(x) = x^3 + mx^2 - nx + 10$

$x - 1$ and $x - 2$ exactly divide $p(x)$

$$\therefore p(1) = 0 \text{ and } p(2) = 0$$

$$p(1) = 1^3 + m \times 1^2 - n \times 1 + 10 = 0$$

$$1 + m - n + 10 = 0$$

$$m - n + 11 = 0$$

$$m - n = -11 \text{ -----(1)}$$

$$m - n = -11 \text{ -----(1)}$$

$$p(2) = 2^3 + m \times 2^2 - n \times 2 + 10 = 0$$

$$8 + 4m - 2n + 10 = 0$$

$$4m - 2n = -18$$

$$2m - n = -9 \text{ ---- \{dividing by 2\}}$$

Subtracting eq. (2) from (1). We get

$$-m = -2$$

$$m = 2$$

Put $m = 2$ in eq. (1). We get

$$2 - n = -11$$

$$-n = -11 - 2$$

$$+n = +13$$

$$n = 13$$

$$m = 2$$

24. Factorise $8a^3 - b^3 - 12a^2b + 6ab^2$

$$\text{Ans. } 8a^3 - b^3 - 12a^2b + 6ab^2$$

$$= (2a)^3 - b^3 - 6ab(2a - b)$$

$$= (2a)^3 - b^3 - 3(2a)(b)(2a - b)$$

$$= (2a - b)^3$$

$$= (2a - b)(2a - b)(2a - b)$$

25. Evaluate $(99)^3$

Ans. $99^3 = (100-1)^3$

We know that $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Take $a = 100$, $b = 1$

$$(100-1)^3 = 100^3 - 3 \times 100^2 \times 1 + 3 \times 100 \times 1^2 - 1^3$$

$$= 1000000 - 300000 + 300 - 1$$

$$= 1000300 - 30001$$

$$= 970299$$

26. Find the value of k, if x-1 is factor of P(x) and $P(x) = 3x^2 + kx + \sqrt{2}$

Ans. x-1 is factor of p(x)

$$\therefore p(1) = 0$$

$$3 \times 1 + k \times 1 + \sqrt{2} = 0$$

$$3 + k + \sqrt{2} = 0$$

$$k = -(3 + \sqrt{2})$$

27. Expand $\left[\frac{2}{3}x + 1\right]^3$

Ans. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$a = \frac{2}{3}x, b = 1$$

$$\left(\frac{2}{3}x + 1\right)^3 = \left(\frac{2}{3}x\right)^3 + 1^3 + 3 \times \frac{2}{3}x \times 1 \left(\frac{2}{3}x + 1\right) = \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2$$

28. Factorise $27x^3 + y^3 + z^3 - 9xyz$

Ans. $27x^3 + y^3 + z^3 - 9xyz$

$$(3x)^3 + (y)^3 + (z)^3 - 3 \times (3x) \times y \times z$$

$$(3x + y + z)[(3x)^2 + (y)^2 + (z)^2 - 3xy - yz - 3xz]$$

$$(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$$

29. Evaluate 105×95

Ans. 105×95

$$= (100+5)(100-5)$$

$$= 100^2 - 5^2 \quad [(a+b)(a-b) = a^2 - b^2]$$

$$= 10000 - 25 = 9975$$

30. Using factor theorem check whether $g(x)$ is factor of $p(x)$

$$p(x) = x^3 - 4x^2 + x + 6, \quad g(x) = x - 3$$

Ans. Given $g(x) = x - 3$, $x - 3 = 0$

Put $x = 3$ in $p(x)$

$$P(3) = 3^3 - 4 \cdot 3^2 + 3 + 6$$

$$= 27 + 9 - 4 \times 9 = 36 - 36 = 0$$

Remainder = 0

\therefore By factor theorem $g(x)$ is factor of $P(x)$

31. Expand $\left(x - \frac{2}{3}y\right)^3$

Ans. $x - \left(\frac{2}{3}y\right)^3$

$$\therefore (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

Hence $a=x$, $b=\frac{2}{3}y$

$$\therefore \left(x - \frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3x \times \frac{2}{3}y \left(x - \frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2xy \left(x - \frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

3 Marks Questions

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Ans. (i) $4x^2 - 3x + 7$

We can observe that in the polynomial $4x^2 - 3x + 7$, we have x as the only variable and the powers of x in each term are a whole number.

Therefore, we conclude that $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

We can observe that in the polynomial $y^2 + \sqrt{2}$, we have y as the only variable and the powers of y in each term are a whole number.

Therefore, we conclude that $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$



We can observe that in the polynomial $3\sqrt{t} + t\sqrt{2}$, we have t as the only variable and the powers of t in each term are not a whole number.

Therefore, we conclude that $3\sqrt{t} + t\sqrt{2}$ is not a polynomial in one variable.

(iv) $y + \frac{2}{y}$

We can observe that in the polynomial $y + \frac{2}{y}$, we have y as the only variable and the powers of y in each term are not a whole number.

Therefore, we conclude that $y + \frac{2}{y}$ is not a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

We can observe that in the polynomial $x^{10} + y^3 + t^{50}$, we have x, y and t as the variables and the powers of x, y and t in each term is a whole number.

Therefore, we conclude that $x^{10} + y^3 + t^{50}$ is a polynomial but not a polynomial in one variable.

2. Write the degree of each of the following polynomials:

(i) $p(x) = 5x^3 + 4x^2 + 7x$

(ii) $p(y) = 4 - y^2$

(iii) $f(t) = 5t - \sqrt{7}$

(iv) $f(x) = 3$

Ans.

(i) $5x^3 + 4x^2 + 7x$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $5x^3 + 4x^2 + 7x$, the highest power of the variable x is 3.

Therefore, we conclude that the degree of the polynomial $5x^3 + 4x^2 + 7x$ is 3.

(ii) $4 - y^2$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $4 - y^2$, the highest power of the variable y is 2.

Therefore, we conclude that the degree of the polynomial $4 - y^2$ is 2.

(iii) $5t - \sqrt{7}$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We observe that in the polynomial $5t - \sqrt{7}$, the highest power of the variable t is 1.

Therefore, we conclude that the degree of the polynomial $5t - \sqrt{7}$ is 1.

(iv) 3

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial 3, the highest power of the assumed variable x is 0.

Therefore, we conclude that the degree of the polynomial 3 is 0.

3. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x-1)(x+1)$

Ans. (i) $p(y) = y^2 - y + 1$

At $p(0)$:

$$p(0) = (0)^2 - 0 + 1 = 1$$

At $p(1)$:

$$p(1) = (1)^2 - 1 + 1 = 1 - 0 = 1$$

At $p(2)$:

$$p(2) = (2)^2 - 2 + 1 = 4 - 1 = 3$$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

At $p(0)$:

$$p(0) = 2 + (0) + 2(0)^2 - (0)^3 = 2$$

At $p(1)$:

$$p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$$

At $p(2)$:

$$p(2) = 2 + (2) + 2(2)^2 - (2)^3 = 4 + 8 - 8 = 4$$

(iii) $p(x) = (x)^3$

At $p(0)$:

$$p(0) = (0)^3 = 0$$

At $p(1)$:

$$p(1) = (1)^3 = 1$$

At $p(2)$:

$$p(2) = (2)^3 = 8$$

(iv) $p(x) = (x-1)(x+1)$

At $p(0)$:

$$p(0) = (0-1)(0+1) = (-1)(1) = -1$$

At $p(1)$:

$$p(1) = (1-1)(2+1) = (0)(3) = 0$$

At $p(2)$:

$$p(2) = (2-1)(2+1) = (1)(3) = 3$$

4. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Ans.

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

We know that according to the factor theorem, $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-1) = 0$.

$$\begin{aligned} p(-1) &= 2(-1)^3 + (-1)^2 - 2(-1) - 1 \\ &= -2 + 1 + 2 - 1 \\ &= 0 \end{aligned}$$

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

We know that according to the factor theorem, $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-2) = 0$.

$$\begin{aligned} p(-2) &= (-2)^3 + 3(-2)^2 + 3(-2) + 1 \\ &= -8 + 12 - 6 + 1 \\ &= -1 \end{aligned}$$

Therefore, we conclude that the $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

We know that according to the factor theorem, $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(3) = 0$.

$$\begin{aligned} p(3) &= (3)^3 - 4(3)^2 + (3) + 6 \\ &= 27 - 36 + 3 + 6 \end{aligned}$$

$$= 0$$

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

5. Find the value of k , if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Ans. (i) $p(x) = x^2 + x + k$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x).$$

We conclude that if $(x - 1)$ is a factor of $p(x) = x^2 + x + k$, then $p(1) = 0$.

$$p(1) = (1)^2 + (1) + k = 0, \text{ or}$$

$$k + 2 = 0$$

$$k = -2$$

Therefore, we can conclude that the value of k is -2 .

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x).$$

We conclude that if $(x - 1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, then $p(1) = 0$.

$$p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0, \text{ or}$$

$$2 + k + \sqrt{2} = 0$$

$$k = -(2 + \sqrt{2}).$$

Therefore, we can conclude that the value of k is $-(2 + \sqrt{2})$.

$$\text{(iii)} \quad p(x) = kx^2 - \sqrt{2}x + 1$$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x).$$

We conclude that if $(x-1)$ is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$, then $p(1) = 0$.

$$p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0, \text{ or}$$

$$k - \sqrt{2} + 1 = 0$$

$$k = \sqrt{2} - 1.$$

Therefore, we can conclude that the value of k is $\sqrt{2} - 1$.

$$\text{(iv)} \quad p(x) = kx^2 - 3x + k$$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x).$$

We conclude that if $(x-1)$ is a factor of $p(x) = kx^2 - 3x + k$, then $p(1) = 0$.

$$p(1) = k(1)^2 - 3(1) + k, \text{ or } 2k - 3 = 0 \Rightarrow k = \frac{3}{2}$$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

6. Factorize:

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

Ans. (i) $12x^2 - 7x + 1$

$$\begin{aligned} 12x^2 - 7x + 1 &= 12x^2 - 3x - 4x + 1 \\ &= 3x(4x - 1) - 1(4x - 1) \\ &= (3x - 1)(4x - 1). \end{aligned}$$

Therefore, we conclude that on factorizing the polynomial $12x^2 - 7x + 1$, we get $(3x - 1)(4x - 1)$.

(ii) $2x^2 + 7x + 3$

$$\begin{aligned} 2x^2 + 7x + 3 &= 2x^2 + 6x + x + 3 \\ &= 2x(x + 3) + 1(x + 3) \\ &= (2x + 1)(x + 3). \end{aligned}$$

Therefore, we conclude that on factorizing the polynomial $2x^2 + 7x + 3$, we get $(2x + 1)(x + 3)$.

(iii) $6x^2 + 5x - 6$

$$\begin{aligned} 6x^2 + 5x - 6 &= 6x^2 + 9x - 4x - 6 \\ &= 3x(2x + 3) - 2(2x + 3) \end{aligned}$$

$$= (3x-2)(2x+3).$$

Therefore, we conclude that on factorizing the polynomial $6x^2 + 5x - 6$, we get $(3x-2)(2x+3)$.

(iv) $3x^2 - x - 4$

$$3x^2 - x - 4 = 3x^2 + 3x - 4x - 4$$

$$= 3x(x+1) - 4(x+1)$$

$$= (3x-4)(x+1).$$

Therefore, we conclude that on factorizing the polynomial $3x^2 - x - 4$, we get $(3x-4)(x+1)$.

7. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$

(ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Ans. (i) $(x+4)(x+10)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(x+4)(x+10)$

$$(x+4)(x+10) = x^2 + (4+10)x + (4 \times 10)$$

$$= x^2 + 14x + 40.$$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2 + 14x + 40$.

(ii) $(x+8)(x-10)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(x+8)(x-10)$

$$\begin{aligned}(x+8)(x-10) &= x^2 + [8+(-10)]x + [8 \times (-10)] \\ &= x^2 - 2x - 80.\end{aligned}$$

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2 - 2x - 80$.

(iii) $(3x+4)(3x-5)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(3x+4)(3x-5)$

$$\begin{aligned}(3x+4)(3x-5) &= (3x)^2 + [4+(-5)]3x + [4 \times (-5)] \\ &= 9x^2 - 3x - 20.\end{aligned}$$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2 - 3x - 20$.

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

We know that $(x+y)(x-y) = x^2 - y^2$.

We need to apply the above identity to find the product $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

$$\begin{aligned}\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) &= (y^2)^2 - \left(\frac{3}{2}\right)^2 \\ &= y^4 - \frac{9}{4}.\end{aligned}$$

Therefore, we conclude that the product $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ is $\left(y^4 - \frac{9}{4}\right)$.

(v) $(3 + 2x)(3 - 2x)$

We know that $(x + y)(x - y) = x^2 - y^2$.

We need to apply the above identity to find the product $(3 + 2x)(3 - 2x)$

$$\begin{aligned}(3 + 2x)(3 - 2x) &= (3)^2 - (2x)^2 \\ &= 9 - 4x^2.\end{aligned}$$

Therefore, we conclude that the product $(3 + 2x)(3 - 2x)$ is $(9 - 4x^2)$.

8. Write the following cubes in expanded form:

(i) $(2x + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x + 1\right)^3$

(iv) $\left(x - \frac{2}{3}y\right)^3$

Ans.

(i) $(2x+1)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\therefore (2x+1)^3 = (2x)^3 + (1)^3 + 3 \times 2x \times 1(2x+1)$$

$$= 8x^3 + 1 + 6x(2x+1)$$

$$= 8x^3 + 12x^2 + 6x + 1.$$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3 + 12x^2 + 6x + 1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\therefore (2a-3b)^3 = (2a)^3 - (3b)^3 - 3 \times 2a \times 3b(2a-3b)$$

$$= 8a^3 - 27b^3 - 18ab(2a-3b)$$

$$= 8a^3 - 36a^2b + 54ab^2 - 27b^3.$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3 - 36a^2b + 54ab^2 - 27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3 \times \frac{3}{2}x \times 1\left(\frac{3}{2}x+1\right) \therefore$$

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$$

$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$.

(iv) $\left(x - \frac{2}{3}y\right)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\begin{aligned}\therefore \left(x - \frac{2}{3}y\right)^3 &= (x)^3 - \left(\frac{2}{3}y\right)^3 - 3 \times x \times \frac{2}{3}y \left(x - \frac{2}{3}y\right) = x^3 - \frac{8}{27}y^3 - 2xy \left(x - \frac{2}{3}y\right) \\ &= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3.\end{aligned}$$

Therefore, the expansion of the expression $\left(x - \frac{2}{3}y\right)^3$ is $x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$.

9. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Ans. (i) $(99)^3$

$(99)^3$ can also be written as $(100-1)^3$.

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$$(100-1)^3 = (100)^3 - (1)^3 - 3 \times 100 \times 1(100-1)$$

$$= 1000000 - 1 - 300(99)$$

$$= 999999 - 29700$$

$$= 970299.$$

(ii) $(102)^3$

$$(102)^3 \text{ can also be written as } (100+2)^3.$$

$$\text{Using identity } (x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

$$(100+2)^3 = (100)^3 + (2)^3 + 3 \times 100 \times 2(100+2)$$

$$= 1000000 + 8 + 600(102)$$

$$= 1000008 + 61200$$

$$= 1061208$$

(iii) $(998)^3$

$$(998)^3 \text{ can also be written as } (1000-2)^3.$$

$$\text{Using identity } (x-y)^3 = x^3 - y^3 - 3xy(x-y)$$

$$(1000-2)^3 = (1000)^3 - (2)^3 - 3 \times 1000 \times 2(1000-2)$$

$$= 1000000000 - 8 - 6000(998)$$

$$= 999999992 - 5988000$$

$$= 994011992$$

10. Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Ans.

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$$= (2a)^3 + (b)^3 + 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$$

$$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b).$$

Using identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ with respect to the expression

$$(2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b), \text{ we get } (2a + b)^3.$$

Therefore, after factorizing the expression $8a^3 + b^3 + 12a^2b + 6ab^2$, we get $(2a + b)^3$.

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as

$$= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$$

$$= (2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression

$$(2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b), \text{ we get } (2a - b)^3.$$

Therefore, after factorizing the expression $8a^3 - b^3 - 12a^2b + 6ab^2$, we get $(2a - b)^3$.

(iii) $27 - 125a^3 - 135a + 225a^2$

The expression $27 - 125a^3 - 135a + 225a^2$ can also be written as

$$= (3)^3 - (5a)^3 - 3 \times 3 \times 3 \times 5a + 3 \times 3 \times 5a \times 5a$$

$$= (3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression $(3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a)$, we get $(3 - 5a)^3$.

Therefore, after factorizing the expression $27 - 125a^3 - 135a + 225a^2$, we get $(3 - 5a)^3$.

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 4a \times 3b + 3 \times 4a \times 3b \times 3b$$

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression $(4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$, we get $(4a - 3b)^3$.

Therefore, after factorizing the expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$, we get $(4a - 3b)^3$.

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

The expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can also be written as

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times 3p \times \frac{1}{6} + 3 \times 3p \times \frac{1}{6} \times \frac{1}{6}$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6} \left(3p - \frac{1}{6}\right).$$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6} \left(3p - \frac{1}{6}\right), \text{ to get } \left(3p - \frac{1}{6}\right)^3.$$

Therefore, after factorizing the expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$, we get $\left(3p - \frac{1}{6}\right)^3$.

11. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2 - 35a + 12$

(ii) Area : $35y^2 + 13y - 12$

Ans.

(i) Area : $25a^2 - 35a + 12$

The expression $25a^2 - 35a + 12$ can also be written as $25a^2 - 15a - 20a + 12$.

$$\begin{aligned} 25a^2 - 15a - 20a + 12 &= 5a(5a - 3) - 4(5a - 3) \\ &= (5a - 4)(5a - 3). \end{aligned}$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2 - 35a + 12$ is Length = $(5a - 4)$ and Breadth = $(5a - 3)$.

(ii) Area : $35y^2 + 13y - 12$

The expression $35y^2 + 13y - 12$ can also be written as $35y^2 + 28y - 15y - 12$.

$$35y^2 + 28y - 15y - 12 = 7y(5y + 4) - 3(5y + 4)$$

$$= (7y - 3)(5y + 4).$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2 + 13y - 12$ is Length = $(7y - 3)$ and Breadth = $(5y + 4)$.

12. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2 - 12x$

(ii) Volume : $12ky^2 + 8ky - 20k$

Ans.

(i) Volume : $3x^2 - 12x$

The expression $3x^2 - 12x$ can also be written as $3 \times x \times (x - 4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2 - 12x$ is 3, x and $(x - 4)$.

(ii) Volume : $12ky^2 + 8ky - 20k$

The expression $12ky^2 + 8ky - 20k$ can also be written as $k(12y^2 + 8y - 20)$.

$$k(12y^2 + 8y - 20) = k(12y^2 - 12y + 20y - 20)$$

$$= k[12y(y - 1) + 20(y - 1)]$$

$$= k(12y + 20)(y - 1)$$

$$= 4k \times (3y + 5) \times (y - 1).$$

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2 + 8ky - 20k$ is $4k$, $(3y+5)$ and $(y-1)$.

13. Using suitable identity expand $\left(\frac{5}{2}x + \frac{3}{4}\right)^3$

Ans. $\left(\frac{5}{2}x + \frac{3}{4}\right)^3$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$\left(\frac{5}{2}x + \frac{3}{4}\right)^3 = \left(\frac{5}{2}x\right)^3 + \left(\frac{3}{4}\right)^3 + 3 \times \frac{5}{2}x \times \frac{3}{4} \left(\frac{5}{2}x + \frac{3}{4}\right)$$

$$= \frac{125x^3}{8} + \frac{27}{64} + \frac{45}{8}x \left(\frac{5}{2}x + \frac{3}{4}\right)$$

$$= \frac{125x^3}{8} + \frac{27}{64} + \frac{225}{16}x^2 + \frac{135}{32}x$$

$$= \frac{125x^3}{8} + \frac{225}{16}x^2 + \frac{135}{32}x + \frac{27}{64}$$

14. Using factor theorem factories $f(x) = x^2 - 5x + 6$

Ans. $f(x) = x^2 - 5x + 6$

Put $x = 1$

$$f(1) = 1^2 - 5 \times 1 + 6 = 2 \neq 0$$

$$\text{Put } x=3 \quad f(2) = 2^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$$

$\therefore x-2$ is factor of $f(x)$

$$\begin{array}{r}
 x-3 \\
 x-2 \overline{) x^2 - 5x + 6} \\
 \underline{x^2 - 2x} \\
 -3x + 6 \\
 \underline{-3x + 6} \\
 0
 \end{array}$$

15. I thought actual division, prove that the polynomial $2x^3 + 4x^2 + x - 34$ is exactly divisible by $(x - 2)$

Ans. Let $f(x) = 2x^3 + 4x^2 + x - 34$

$x - 2$ Is factor of $f(x)$

$x = 2$ Zero of $f(x)$

$$f(2) = 2 \times 2^3 + 4 \times 2^2 + 2 - 34$$

$$= 16 + 16 + 2 - 34$$

$$= 34 - 34 = 0$$

$2x^3 + 4x^2 + x - 34$ is divisible by $x - 2$

16. Factorise $1 - a^2 - b^2 - 2ab$

Ans. $1 - a^2 - b^2 - 2ab$

$$1 - (a^2 + b^2 + 2ab) = 1^2 - (a + b)^2$$

$$= (1 + a + b)(1 - a - b)$$

17. Expand $\left(\frac{1}{2}a - \frac{1}{3}b + 1\right)^2$

Ans. $\left(\frac{1}{2}a - \frac{1}{3}b + 1\right)^2$

$$\begin{aligned} &= \left(\frac{1}{2}a\right)^2 + \left(-\frac{1}{3}b\right)^2 + 1^2 + 2 \times \frac{1}{2}a \times \left(-\frac{1}{3}b\right) + 2 \times \left(-\frac{1}{3}b\right) \times 1 + 2 \times \left(\frac{1}{2}a\right) \times 1 \\ &= \frac{a^2}{4} + \frac{b^2}{9} + 1 - \frac{ab}{3} - \frac{2b}{3} + a \end{aligned}$$

18. Verify each of the following identities

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Ans. (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Taking R.H.S

$$\begin{aligned} &(x + y)(x^2 - xy + y^2) \\ &= x(x^2 - xy + y^2) + y(x^2 - xy + y^2) \\ &= x^3 - \cancel{xy^2} + \cancel{xy^2} + \cancel{yx^2} - \cancel{xy^2} + y^3 \\ &= x^3 + y^3 = L.H.S. \end{aligned}$$

$$L.H.S = R.H.S.$$

Verified

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$$R.H.S = x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$$

$$= x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{yx^2} - \cancel{xy^2} - y^3$$

$$= x^3 - y^3$$

= L.H.S.

$$L.H.S. = R.H.S.$$

Verified

19. Using identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ derive the formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Ans. given $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\therefore a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$= (a + b)[(a + b)^2 - 3ab]$$

$$= (a + b)[a^2 + b^2 + 2ab - 3ab]$$

$$= (a + b)(a^2 + b^2 - ab)$$

$$= (a + b)(a^2 - ab + b^2)$$

20. Factories

(i) $64y^3 + 125z^3$

(ii) $27m^3 - 343n^3$

Ans. Solution

(i) $64y^3 + 125z^3$

$$(4y)^3 + (5z)^3$$

$$(4y + 5z)[(4y)^2 - 4y \times 5z + (5z)^2]$$

$$[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= (4y + 5z)(16y^2 - 20yz + 25z^2)$$

$$\text{(ii) } 27m^3 - 343n^3$$

$$= (3m)^3 - (7n)^3$$

$$= (3m - 7n)[(3m)^2 + 3m \times 7n + (7n)^2]$$

$$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$(3m - 7n)(9m^2 + 21mn + 49n^2)$$

21. Without actually calculating the cubes. Find the value of $(26)^3 + (-15)^3 + (11)^3$

Ans. Let $a = 26$, $b = -15$, $c = -11$

$$a + b + c = 26 - 15 - 11 = 0$$

$$\text{Then } a^3 + b^3 + c^3 = 3abc$$

$$(26)^3 + (-15)^3 + (-11)^3$$

$$= 3 \times 26 \times -15 \times -11$$

$$= 12870$$

22. Find the values of m and n so that the polynomial $x^3 - mx^2 - 13x + n$ has $x-1$ and $x+3$ as factors.

Ans. Let polynomial be

$$p(x) = x^3 - mx^2 - 13x + n$$

If $x-1$ is factor of $p(x)$

$$\therefore p(1) = 0$$

$$(1)^3 - m(1)^2 - 13 \times 1 + n = 0$$

$$1 - m - 13 + n = 0$$

$$-m + n - 12 = 0$$

$$-12 = m - n \dots\dots\dots(1)$$

And if $x-3$ is factor of $p(x)$

$$\therefore p(-3) = 0$$

$$(-3)^3 - m(-3)^2 - 13 \times (-3) + n = 0$$

$$-27 - 9m + 39 + n = 0$$

$$-9m + n + 12 = 0$$

$$12 = 9m - n$$

$$12 = 9m - n$$

Subtracting (1) from (2),

$$8m = 24$$

$$m = \frac{24}{8}$$

$$m = 3$$

Put $m = 3$ in (1),

$$3 - n = -12$$

$$-n = -12 - 3$$

$$-n = -15$$

$$N = 15$$

$$\therefore m = 3 \text{ and } n = 15$$

23. Prove that $x^2 + 6x + 15$ has no zero.

Ans. $x^2 + 6x + 15$

$$= x^2 + 2 \times 3x + 3^2 + 6$$

$$= (x+3)^2 + 6$$

$(x+3)^2$ is positive and 6 is positive

$\therefore (x+3)^2 + 6$ has no zero.

$x^2 + 6x + 15$ has no zero.

24. Factorise $3(x+y)^2 - 5(x+y) + 2$

Ans. $3(x+y)^2 - 5(x+y) + 2$

Let $x+y = z$

$$= 3z(z-1) - 2(z-1)$$

$$= (3z-2)(z-1)$$

Put $z = x+y$

$$\therefore 3(x+y)^2 - 5(x+y) + 2$$

$$= [3(x+y) - 2] [x+y - 1]$$

$$= [3x+3y - 2] [x+y - 1]$$

$$=3z^2 - 5z + 2$$

25. The volume of a cuboid is given by the expression $3x^3-12x$. Find the possible expressions for its dimensions

Ans. The volume of cuboid is given by

$$3x^3 - 12x = 3x(x^2 - 4) = 3x(x+2)(x-2)$$

Dimensions of the cuboid are given by $3x$, $(x+2)$ and $(x-2)$

$$P(1) = 1^3 - m \times 1^2 - 13 \times 1 + n = 0$$

$$= 1 - m - 13 + n = 0$$

$$= -m + n = 12 \quad (1)$$

$x+3$ is factor of $P(x)$

$$\therefore P(-3) = 0$$

$$P(-3) = (-3)^3 - m(-3)^2 - 13 \times (-3) + n = 0$$

$$= -27 - 9m + 39 + n = 0$$

$$= -9m + n + 12 = 0 \quad (2)$$

$$= -9m + n = -12$$

Subtracting eq. (2) from (1)

$$8m = 24, m = 3$$

Put $m = 3$ in eq(1)

$$m = 3 \text{ and } n = 15$$

26. Using remainder theorem factories

$$x^3 - 3x^2 - x + 3$$

Ans. $x^3 - 3x^2 - x + 3$

Coefficient of x^3 is 1

Constant = 3

$$3 \times 1 = 3$$

\therefore We can Put $x = \pm 3$ and (\bar{x}) and check

Put $x = 1$

$$1^3 - 3 \times 1^2 - 1 + 3$$

$$1 - 3 - 1 + 3 = 0$$

Remainder = 0

$\therefore x - 1$ is factor of $x^3 - 3x^2 - x + 3$

$$\begin{array}{r} x-1 \overline{) \begin{array}{r} x^3 - 3x^2 + 3 \\ x^3 - x^2 \\ \hline -2x^2 - x + 3 \\ -2x^2 + 2x \\ \hline -3x + 3 \\ -3x + 3 \\ \hline 0 \end{array}} \end{array}$$

$$\therefore x^3 - 3x^2 - x + 3 = (x-1)(x^2 - 2x - 3)$$

$$= (x-1)(x^2 - 3x + x - 3)$$

$$= (x-1)[x(x-3) + 1(x-3)]$$

$$= (x-1)(x-3)(x+1)$$

27. If $y^3 + ay^2 + by + 6$ is divisible by $y - 2$ and leaves remainder 3 when divided by $y - 3$, find the values of a and b .

Ans. Let

$$p(y) = y^3 + ay^2 + by + 6$$

$p(y)$ is divisible by $y - 2$

Then $P(2) = 0$

$$2^3 + a \times 2^2 + b \times 2 + 6 = 0$$

$$8 + 4a + 2b + 6 = 0$$

$$4a + 2b = -14$$

$$2a + b = -7 \quad (i)$$

If $p(y)$ is divided by $y - 3$ remainder is 3

$$\therefore p(3) = 3$$

$$3^3 + a \times 3^2 + b \times 3 + 6 = 3$$

$$9a + 3b = -30$$

$$3a + b = -10 \quad \text{---(ii)}$$

Subtracting (i) from (ii)

$$-a = 3 \text{ and } a = -3$$

Put $a = -3$ in eq (i)

$$2 \times -3 + b = -7$$

$$-6 + b = -7$$

$$b = -7 + 6$$

B=-1

28. Factorise $x^6 - 64$

Ans. $x^6 - 64$

$$= (x^2)^3 - (2^2)^3$$

$$= (x^2 - 2^2) [x^4 + 4x^2 + 16]$$

$$= (x+2) (x-2) (x^4 + 4x^2 + 16)$$

29. The volume of a cuboid is given by the algebraic expression $ky^2 - 6ky + 8k$. Find the possible expressions for the dimensions of the cuboid.

Ans. Given volume of cuboid

$$ky^2 - 6ky + 8k$$

$$= k [y^2 - 6y + 8]$$

$$k [y^2 - 4y - 2y + 8]$$

$$= k [y(y-4) - 2(y-4)] = k (y-2) (y-4)$$

Thus dimension of cuboid

k, (y-2) and (y-4)

CBSE Class 9 Mathemaics
Important Questions
Chapter 2
Polynomials

4 Marks Questions

1. Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2 + x$

(ii) $x - x^3$

(iii) $y + y^2 + 4$

(iv) $1 + x$

(v) $3t$

(vi) r^2

(vii) $7x^3$

Ans.

(i) $x^2 + x$

We can observe that the degree of the polynomial $x^2 + x$ is 2.

Therefore, we can conclude that the polynomial $x^2 + x$ is a quadratic polynomial.

(ii) $x - x^3$

We can observe that the degree of the polynomial $x - x^3$ is 3.

Therefore, we can conclude that the polynomial $x - x^3$ is a cubic polynomial.

(iii) $y + y^2 + 4$



We can observe that the degree of the polynomial $y + y^2 + 4$ is 2.

Therefore, the polynomial $y + y^2 + 4$ is a quadratic polynomial.

(iv) $1 + x$

We can observe that the degree of the polynomial $(1 + x)$ is 1.

Therefore, we can conclude that the polynomial $1 + x$ is a linear polynomial.

(v) $3t$

We can observe that the degree of the polynomial $(3t)$ is 1.

Therefore, we can conclude that the polynomial $3t$ is a linear polynomial.

(vi) r^2

We can observe that the degree of the polynomial r^2 is 2.

Therefore, we can conclude that the polynomial r^2 is a quadratic polynomial.

(vii) $7x^3$

We can observe that the degree of the polynomial $7x^3$ is 3.

Therefore, we can conclude that the polynomial $7x^3$ is a cubic polynomial.

2. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$

(ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, x = -1, 1$

(iv) $p(x) = (x + 1)(x - 2), x = -1, 2$

(v) $p(x) = x^2, \quad x = 0$

(vi) $p(x) = lx + m, \quad x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1, \quad x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1, \quad x = -\frac{1}{2}$

Ans. (i) $p(x) = 3x + 1, \quad x = -\frac{1}{3}$

We need to check whether $p(x) = 3x + 1$ at $x = -\frac{1}{3}$ is equal to zero or not.

$$p\left(-\frac{1}{3}\right) = 3x + 1 = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

Therefore, we can conclude that $x = -\frac{1}{3}$ is a zero of the polynomial $p(x) = 3x + 1$.

(ii) $p(x) = 5x - \pi, \quad x = \frac{4}{5}$

We need to check whether $p(x) = 5x - \pi$ at $x = \frac{4}{5}$ is equal to zero or not.

$$p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$$

Therefore, $x = \frac{4}{5}$ is not a zero of the polynomial $p(x) = 5x - \pi$.

(iii) $p(x) = x^2 - 1, \quad x = -1, 1$

We need to check whether $p(x) = x^2 - 1$ at $x = -1, 1$ is equal to zero or not.

At $x = -1$

$$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$$

At $x = 1$

$$p(1) = (1)^2 - 1 = 1 - 1 = 0$$

Therefore, $x = -1, 1$ are the zeros of the polynomial $p(x) = x^2 - 1$.

(iv) $p(x) = (x+1)(x-2)$, $x = -1, 2$

We need to check whether $p(x) = (x+1)(x-2)$ at $x = -1, 2$ is equal to zero or not.

At $x = -1$

$$p(-1) = (-1+1)(-1-2) = (0)(-3) = 0$$

At $x = 2$

$$p(2) = (2+1)(2-2) = (3)(0) = 0$$

Therefore, $x = -1, 2$ are the zeros of the polynomial $p(x) = (x+1)(x-2)$.

(v) $p(x) = x^2$, $x = 0$

We need to check whether $p(x) = x^2$ at $x = 0$ is equal to zero or not.

$$p(0) = (0)^2 = 0$$

Therefore, we can conclude that $x = 0$ is a zero of the polynomial $p(x) = x^2$.

(vi) $p(x) = lx + m$, $x = -\frac{m}{l}$

We need to check whether $p(x) = lx + m$ at $x = -\frac{m}{l}$ is equal to zero or not.

$$p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = m + m = 0$$

Therefore, $x = -\frac{m}{l}$ is a zero of the polynomial $p(x) = lx + m$.

$$\text{(vii)} \quad p(x) = 3x^2 - 1, \quad x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

We need to check whether $p(x) = 3x^2 - 1$ at $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ is equal to zero or not.

$$\text{At } x = -\frac{1}{\sqrt{3}}$$

$$p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$$

$$\text{At } x = \frac{2}{\sqrt{3}}$$

$$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$$

Therefore, we can conclude that $x = -\frac{1}{\sqrt{3}}$ is a zero of the polynomial $p(x) = 3x^2 - 1$ but

$x = \frac{2}{\sqrt{3}}$ is not a zero of the polynomial $p(x) = 3x^2 - 1$.

$$\text{(viii)} \quad p(x) = 2x + 1, \quad x = -\frac{1}{2}$$

We need to check whether $p(x) = 2x + 1$ at $x = -\frac{1}{2}$ is equal to zero or not.

$$p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) + 1 = -1 + 1 = 0$$

Therefore, $x = -\frac{1}{2}$ is a zero of the polynomial $p(x) = 2x + 1$

3. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$

(ii) $x - \frac{1}{2}$

(iii) x

(iv) $x + \pi$

(v) $5 + 2x$

Ans.

(i) $x + 1$

We need to find the zero of the polynomial $x + 1$.

$$x + 1 = 0 \quad \Rightarrow \quad x = -1$$

While applying the remainder theorem, we need to put the zero of the polynomial $x + 1$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p(0) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$$

$$= -1 + 3 - 3 + 1$$

$$= 0$$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by $x + 1$, we will

get the remainder as 0.

(ii) $x - \frac{1}{2}$

We need to find the zero of the polynomial $x - \frac{1}{2}$.

$$x - \frac{1}{2} = 0$$

$$\Rightarrow x = \frac{1}{2}$$

While applying the remainder theorem, we need to put the zero of the polynomial $x - \frac{1}{2}$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + 3\left(\frac{1}{4}\right) + \frac{3}{2} + 1$$

$$= \frac{1+6+12+8}{8}$$

$$= \frac{27}{8}$$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by $x - \frac{1}{2}$, we will get the remainder as $\frac{27}{8}$.

(iii) x

We need to find the zero of the polynomial x .

$$x = 0$$

While applying the remainder theorem, we need to put the zero of the polynomial x in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p(0) = (0)^3 + 3(0)^2 + 3(0) + 1$$

$$= 0 + 0 + 0 + 1$$

$$= 1$$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by x , we will get the remainder as 1.

(iv) $x + \pi$

We need to find the zero of the polynomial $x + \pi$.

$$x + \pi = 0$$

$$\Rightarrow x = -\pi$$

While applying the remainder theorem, we need to put the zero of the polynomial $x + \pi$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$$

$$= -\pi^3 + 3\pi^2 - 3\pi + 1.$$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by $x + \pi$, we will get the remainder as $-\pi^3 + 3\pi^2 - 3\pi + 1$.

(v) $5 + 2x$

We need to find the zero of the polynomial $5 + 2x$.

$$5 + 2x = 0$$

$$\Rightarrow x = -\frac{5}{2}$$

While applying the remainder theorem, we need to put the zero of the polynomial $5 + 2x$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$$

$$= -\frac{125}{8} + 3\left(\frac{25}{4}\right) - \frac{15}{2} + 1$$

$$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1$$

$$= \frac{-125 + 150 - 60 + 8}{8}$$

$$= -\frac{27}{4}.$$

4. Determine which of the following polynomials has $(x+1)$ a factor:

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Ans. (i) $x^3 + x^2 + x + 1$

While applying the factor theorem, we get

$$p(x) = x^3 + x^2 + x + 1$$

$$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$

$$= -1 + 1 - 1 + 1$$

$$= 0$$

We conclude that on dividing the polynomial $x^3 + x^2 + x + 1$ by $(x + 1)$, we get the remainder as 0.

Therefore, we conclude that $(x + 1)$ is a factor of $x^3 + x^2 + x + 1$.

(ii) $x^4 + x^3 + x^2 + x + 1$

While applying the factor theorem, we get

$$p(x) = x^4 + x^3 + x^2 + x + 1$$

$$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

$$= 1 - 1 + 1 - 1 + 1$$

$$= 1$$

We conclude that on dividing the polynomial $x^4 + x^3 + x^2 + x + 1$ by $(x + 1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

While applying the factor theorem, we get

$$p(x) = x^4 + 3x^3 + 3x^2 + x + 1$$

$$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$= 1 - 3 + 3 - 1 + 1$$

$$= 1$$

We conclude that on dividing the polynomial $x^4 + 3x^3 + 3x^2 + x + 1$ by $(x+1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

$$(iv) x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$$

While applying the factor theorem, we get

$$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$$

$$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$$

$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}.$$

We conclude that on dividing the polynomial $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ by $(x+1)$, we will get the remainder as $2\sqrt{2}$, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

5. Expand each of the following, using suitable identities:

$$(i) (2x - y + z)^2$$

$$(ii) (-2x + 3y + 2z)^2$$

(iii) $(3a - 7b - c)^2$

(iv) $(-2x + 5y - 3z)^2$

(v) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

Ans.

(i) $(2x - y + z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(2x - y + z)^2$.

$$\begin{aligned}(2x - y + z)^2 &= [2x + (-y) + z]^2 \\&= (2x)^2 + (-y)^2 + (z)^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times z + 2 \times z \times 2x \\&= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx.\end{aligned}$$

(ii) $(-2x + 3y + 2z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x + 3y + 2z)^2$.

$$\begin{aligned}(-2x + 3y + 2z)^2 &= [(-2x) + 3y + 2z]^2 \\&= (-2x)^2 + (3y)^2 + (2z)^2 + 2 \times (-2x) \times 3y + 2 \times 3y \times 2z + 2 \times 2z \times (-2x) \\&= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx.\end{aligned}$$

(iii) $(3a - 7b - c)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(3a - 7b - c)^2$.

$$\begin{aligned}(3a - 7b - c)^2 &= [3a + (-7b) + (-c)]^2 \\&= (3a)^2 + (-7b)^2 + (-c)^2 + 2 \times 3a \times (-7b) + 2 \times (-7b) \times (-c) + 2 \times (-c) \times 3a \\&= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac.\end{aligned}$$

(iv) $(-2x + 5y - 3z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x + 5y - 3z)^2$.

$$\begin{aligned}(-2x + 5y - 3z)^2 &= [(-2x) + 5y + (-3z)]^2 \\&= (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \times (-2x) \times 5y + 2 \times 5y \times (-3z) + 2 \times (-3z) \times (-2x) \\&= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx.\end{aligned}$$

(v) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

$$\begin{aligned}\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 &= \left[\frac{a}{4} + \left(-\frac{b}{2}\right) + 1\right]^2 \\&= \left(\frac{a}{4}\right)^2 + \left(-\frac{b}{2}\right)^2 + (1)^2 + 2 \times \frac{a}{4} \times \left(-\frac{b}{2}\right) + 2 \times \left(-\frac{b}{2}\right) \times 1 + 2 \times 1 \times \frac{a}{4}\end{aligned}$$

$$= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}.$$

6. Factorize:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Ans.

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

The expression $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ can also be written as

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x.$$

We can observe that, we can apply the identity

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx \text{ with respect to the expression}$$

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x, \text{ to get}$$

$$(2x + 3y - 4z)^2$$

Therefore, we conclude that after factorizing the expression

$$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz, \text{ we get } (2x + 3y - 4z)^2.$$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

We need to factorize the expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$.

The expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$ can also be written as

$$(-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x) \times y + 2 \times y \times (2\sqrt{2}z) + 2 \times (2\sqrt{2}z) \times (-\sqrt{2}x).$$

We can observe that, we can apply the identity

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ with respect to the expression
 $(-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x) \times y + 2 \times y \times (2\sqrt{2}z) + 2 \times (2\sqrt{2}z) \times (-\sqrt{2}x),$
 to get

$$(-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

Therefore, we conclude that after factorizing the expression

$$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz, \text{ we get } (-\sqrt{2}x + y + 2\sqrt{2}z)^2.$$

CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

5 Marks Questions

1. Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$

(vi) $p(x) = ax, a \neq 0$

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

Ans.

(i) $p(x) = x + 5$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = x + 5$ equal to 0, we get

$$x + 5 = 0$$

$$\Rightarrow x = -5$$

Therefore, we conclude that the zero of the polynomial $p(x) = x + 5$ is -5 .

(ii) $p(x) = x - 5$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = x - 5$ equal to 0, we get

$$x - 5 = 0$$

$$\Rightarrow x = 5$$

Therefore, we conclude that the zero of the polynomial $p(x) = x - 5$ is 5.

(iii) $p(x) = 2x + 5$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = 2x + 5$ equal to 0, we get

$$2x + 5 = 0$$

$$\Rightarrow x = \frac{-5}{2}$$

Therefore, we conclude that the zero of the polynomial $p(x) = 2x + 5$ is $\frac{-5}{2}$.

(iv) $p(x) = 3x - 2$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = 3x - 2$ equal to 0, we get

$$3x - 2 = 0$$

$$\Rightarrow x = \frac{2}{3}$$

Therefore, we conclude that the zero of the polynomial $p(x) = 3x - 2$ is $\frac{2}{3}$.

(v) $p(x) = 3x$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = 3x$ equal to 0, we get

$$3x = 0$$

$$\Rightarrow x = 0$$

Therefore, we conclude that the zero of the polynomial $p(x) = 3x$ is 0.

(vi) $p(x) = ax, a \neq 0$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = ax$ equal to 0, we get

$$ax = 0$$

$$\Rightarrow x = 0$$

Therefore, we conclude that the zero of the polynomial $p(x) = ax, a \neq 0$ is 0.

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = cx + d$ equal to 0, we get

$$cx + d = 0$$

$$\Rightarrow x = -\frac{d}{c}.$$

Therefore, we conclude that the zero of the polynomial

$$p(x) = cx + d, c \neq 0, c, d \text{ are real numbers is } -\frac{d}{c}.$$

2. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Ans. We know that if the polynomial $7 + 3x$ is a factor of $3x^3 + 7x$, then on dividing the polynomial $3x^3 + 7x$ by $7 + 3x$, we must get the remainder as 0.

We need to find the zero of the polynomial $7 + 3x$.

$$7 + 3x = 0$$

$$\Rightarrow x = -\frac{7}{3}$$

While applying the remainder theorem, we need to put the zero of the polynomial $7 + 3x$ in the polynomial $3x^3 + 7x$, to get

$$p(x) = 3x^3 + 7x$$

$$= 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right) = 3\left(-\frac{343}{27}\right) - \frac{49}{3}$$

$$= -\frac{343}{9} - \frac{49}{3} = \frac{-343 - 147}{9}$$

$$= \frac{-490}{9}.$$

We conclude that on dividing the polynomial $3x^3 + 7x$ by $7 + 3x$, we will get the remainder

as $\frac{-490}{9}$, which is not 0.

Therefore, we conclude that $7 + 3x$ is not a factor of $3x^3 + 7x$.

3. Factorize:

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Ans.

(i) $x^3 - 2x^2 - x + 2$

We need to consider the factors of 2, which are $\pm 1, \pm 2$.

Let us substitute 1 in the polynomial $x^3 - 2x^2 - x + 2$, to get

$$(1)^3 - 2(1)^2 - (1) + 2 = 1 - 2 - 1 + 2 = 0$$

Thus, according to factor theorem, we can conclude that $(x - 1)$ is a factor of the polynomial $x^3 - 2x^2 - x + 2$.

Let us divide the polynomial $x^3 - 2x^2 - x + 2$ by $(x - 1)$, to get

$$\begin{array}{r}
 x^2 - x - 2 \\
 x-1 \overline{) x^3 - 2x^2 - x + 2} \\
 \underline{x^3 - x^2} \\
 -x^2 - x \\
 \underline{-x^2 + x} \\
 -2x + 2 \\
 \underline{-2x + 2} \\
 0
 \end{array}$$

$$x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2).$$

$$x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2)$$

$$= (x-1)(x^2 + x - 2x - 2)$$

$$= (x-1)[x(x+1) - 2(x+1)]$$

$$= (x-1)(x-2)(x+1).$$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 2x^2 - x + 2$, we get $(x-1)(x-2)(x+1)$.

(ii) $x^3 - 3x^2 - 9x - 5$

We need to consider the factors of -5 , which are $\pm 1, \pm 5$.

Let us substitute 1 in the polynomial $x^3 - 3x^2 - 9x - 5$, to get

$$(-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = 0$$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3 - 3x^2 - 9x - 5$.

Let us divide the polynomial $x^3 - 3x^2 - 9x - 5$ by $(x+1)$, to get

$$\begin{array}{r}
 x^2 - 4x - 5 \\
 x+1 \overline{) x^3 - 3x^2 - 9x - 5} \\
 \underline{x^3 + x^2} \\
 -4x^2 - 9x \\
 \underline{-4x^2 - 4x} \\
 -5x - 5 \\
 \underline{-5x - 5} \\
 0
 \end{array}$$

$$x^3 - 3x^2 - 9x - 5 = (x+1)(x^2 - 4x - 5)$$

$$= (x+1)(x^2 + x - 5x - 5)$$

$$= (x+1)[x(x+1) - 5(x+1)]$$

$$= (x+1)(x-5)(x+1).$$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 3x^2 - 9x - 5$, we get $(x+1)(x-5)(x+1)$.

(iii) $x^3 + 13x^2 + 32x + 20$

We need to consider the factors of 20, which are $\pm 5, \pm 4, \pm 2, \pm 1$.

Let us substitute -1 in the polynomial $x^3 + 13x^2 + 32x + 20$, to get

$$(-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = -20 + 20 = 0$$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3 + 13x^2 + 32x + 20$.

Let us divide the polynomial $x^3 + 13x^2 + 32x + 20$ by $(x+1)$, to get

$$\begin{array}{r}
 x^2 + 12x + 20 \\
 x+1 \overline{) x^3 + 13x^2 + 32x + 20} \\
 \underline{x^3 + x^2} \\
 12x^2 + 32x \\
 \underline{12x^2 + 12x} \\
 20x + 20 \\
 \underline{20x + 20} \\
 0
 \end{array}$$

$$\begin{aligned}
 x^3 + 13x^2 + 32x + 20 &= (x+1)(x^2 + 12x + 20) \\
 &= (x+1)(x^2 + 2x + 10x + 20) \\
 &= (x+1)[x(x+2) + 10(x+2)] \\
 &= (x+1)(x+10)(x+2).
 \end{aligned}$$

Therefore, we can conclude that on factorizing the polynomial $x^3 + 13x^2 + 32x + 20$, we get $(x+1)(x-10)(x+2)$.

(iv) $2y^3 + y^2 - 2y - 1$

We need to consider the factors of -1 , which are ± 1 .

Let us substitute 1 in the polynomial $2y^3 + y^2 - 2y - 1$, to get

$$2(1)^3 + (1)^2 - 2(1) - 1 = 2 + 1 - 2 - 1 = 3 - 3 = 0$$

Thus, according to factor theorem, we can conclude that $(y-1)$ is a factor of the polynomial $2y^3 + y^2 - 2y - 1$.

Let us divide the polynomial $2y^3 + y^2 - 2y - 1$ by $(y-1)$, to get

$$\begin{array}{r}
 2y^2 + 3y + 1 \\
 y-1 \overline{) 2y^3 + y^2 - 2y - 1} \\
 \underline{2y^3 - 2y^2} \\
 3y^2 - 2y \\
 \underline{3y^2 - 3y} \\
 y - 1 \\
 \underline{y - 1} \\
 0
 \end{array}$$

$$2y^3 + y^2 - 2y - 1 = (y-1)(2y^2 + 3y + 1)$$

$$= (y-1)(2y^2 + 2y + y + 1)$$

$$= (y-1)[2y(y+1) + 1(y+1)]$$

$$= (y-1)(2y+1)(y+1).$$

Therefore, we can conclude that on factorizing the polynomial $2y^3 + y^2 - 2y - 1$, we get $(y-1)(2y+1)(y+1)$.

4. If $x^2 - bx + c = (x + p)(x - q)$ then factories $x^2 - bxy + cy^2$

Ans. We have $x^2 - bx + c = (x + p)(x - q)$

$$x^2 - bx + c = x^2 + (p - q)x - pq$$

Equating coefficient of x and constant

$$-b = p - q \text{ and } c = -pq$$

Substituting these values of b and c in $x^2 - bxy + cy^2$, We get

$$x^2 + (p - q)xy - pqy^2$$

$$x^2 + pxy - qxy - pqy^2$$

$$x(x + py) - qy(x + py)$$

$$(x + py)(x - qy)$$

5. Factorise $(2x - 3y)^3 + (3y - 4z)^3 + (4z - 2x)^3$

Ans. Let $a = 2x - 3y$, $b = 3y - 4z$, $c = 4z - 2x$

then $a + b + c = \cancel{2x} - \cancel{3y} + \cancel{3y} - \cancel{4z} + \cancel{4z} - \cancel{2x} = 0$

$$\therefore a^3 + b^3 + c^3 = 3abc$$

$$(2x - 3y)^3 + (3y - 4z)^3 + (4z - 2x)^3 = 3(2x - 3y)(3y - 4z)(4z - 2x)$$

$$= 3(2x - 3y)(3y - 4z) \times 2(2z - x)$$

$$= 6(2x - 3y)(3y - 4z)(2z - x)$$

6. Factorise: $12(y^2 + 7y)^2 - 8(y^2 + 7y)(2y - 1) - 15(2y - 1)^2$

Ans. Let $a = y^2 + 7y$, $b = 2y - 1$

Then $12(y^2 + 7y)^2 - 8(y^2 + 7y)(2y - 1) - 15(2y - 1)^2$

$$= 12a^2 - 8ab - 15b^2$$

$$= 12a^2 - 18ab + 10ab - 15b^2$$

$$= 6a(2a - 3b) + 5b(2a - 3b)$$

$$= (2a - 3b)(6a + 5b)$$

Put $a = y^2 + 7y$ and $b = 2y - 1$

$$= [2(y^2 + 7y) - 3(2y - 1)][6(y^2 + 7y) + 5(2y - 1)]$$

$$= [2y^2 + 14y - 6y + 3][6y^2 + 42y + 10y - 5]$$

$$= (2y^2 + 8y + 3)(6y^2 + 52y - 5)$$

7. Factorise $x^6 + 8y^6 - z^6 + 6x^2y^2z^2$

Ans. $x^6 + 8y^6 - z^6 + 6x^2y^2z^2$

$$= (x^2)^3 + (2y^2)^3 + (-z^2)^3 - 3(x^2)(2y^2)(-z^2)$$

$$= [x^2 + 2y^2 - z^2][(x^2)^2 + (2y^2)^2 + (-z^2)^2 - x^2 \times 2y^2 - 2y^2(-z^2) - x^2 \times (-z^2)^2]$$

$$= (x^2 + 2y^2 - z^2)(x^4 + 4y^4 + z^4 - 2x^2y^2 + 2y^2z^2 + x^2z^2)$$

8. Factorise: $\frac{1}{27}(2x+5y)^3 + \left(\frac{-5}{3}y + \frac{3}{4}z\right)^3 - \left(\frac{3}{4}z + \frac{2}{3}x\right)^3$

Ans. Given expression can be written as

$$\left[\frac{1}{3}(2x+5y)\right]^3 + \left[-\frac{5}{3}y + \frac{3}{4}z\right]^3 + \left[-\frac{3}{4}z - \frac{2}{3}x\right]^3$$

$$\text{Let } \frac{1}{3}(2x+5y) = a, \frac{-5}{3}y + \frac{3}{4}z = b$$

$$\text{and } \frac{-3}{4}z - \frac{2}{3}x = c$$

$$a + b + c = \frac{2}{3}x + \frac{5}{3}y - \frac{5}{3}y + \frac{5}{4}z - \frac{3}{4}z - \frac{2}{3}x = 0$$

$$\therefore a^3 + b^3 + c^3 = 3abc$$

Thus,

$$\begin{aligned}& \frac{1}{27} (2x+5y)^3 + \left(\frac{-5}{3}y + \frac{3}{4}z \right)^3 - \left(\frac{3}{4}z + \frac{2}{3}x \right)^3 \\&= 3 \left[\frac{1}{3} (2x+5y) \left(\frac{-5}{3}y + \frac{3}{4}z \right) \left(\frac{-3}{4}z - \frac{2}{3}x \right) \right] \\&= -(2x+5y) \left(\frac{-5}{3}y + \frac{3}{4}z \right) \left(\frac{3}{4}z + \frac{2}{3}x \right) \\&= -(2x+5y) \left(\frac{-20y+9z}{12} \right) \left(\frac{9z+8x}{12} \right) \\&= \frac{1}{144} (2x+5y) (20y-9z) (9z+8x)\end{aligned}$$